Property	Time Domain	Frequency Domain
Linearity	$a_1 x_1(n) + a_2 x_2(n)$	$a_1 X_1(\Omega) + a_2 X_2(\Omega)$
Translation	$x(n - n_0)$	$e^{-j\Omega n_0} X(\Omega)$
Modulation	$e^{j\Omega_0 n} x(n)$	$X(\Omega - \Omega_0)$
Time Reversal	$x(-n)$	$X(-\Omega)$
Conjugation	$x^*(n)$	$X^*(-\Omega)$
Downsampling	$x(Mn)$	$\frac{1}{M}\sum_{k=0}^{M-1} X\left(\frac{\Omega - 2\pi k}{M}\right)$
Upsampling	$(\uparrow M)x(n)$	$X(M\Omega)$
Convolution	$x_1 * x_2(n)$	$X_1(\Omega) X_2(\Omega)$
Multiplication	$x_1(n) x_2(n)$	$\frac{1}{2\pi}\int_{2\pi} X_1(\theta) X_2(\Omega - \theta)\, d\theta$
Freq.-Domain Diff.	$nx(n)$	$j\frac{d}{d\Omega} X(\Omega)$
Accumulation	$\sum_{k=-\infty}^{n} x(k)$	$\frac{e^{j\Omega}}{e^{j\Omega} - 1} X(\Omega) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$

Property	
Periodicity	$X(\Omega) = X(\Omega + 2\pi)$
Parseval's Relation	$\sum_{n=-\infty}^{\infty} \lvert x(n) \rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(\Omega) \rvert^2 d\Omega$

Pair	$x(n)$	$X(\Omega)$
1	$\delta(n)$	1
2	1	$2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$
3	$u(n)$	$\frac{e^{j\Omega}}{e^{j\Omega}-1}+\sum_{k=-\infty}^{\infty}\pi\delta(\Omega-2\pi k)$
4	$a^n u(n), \lvert a\rvert < 1$	$\frac{e^{j\Omega}}{e^{j\Omega}-a}$
5	$-a^n u(-n-1), \lvert a\rvert > 1$	$\frac{e^{j\Omega}}{e^{j\Omega}-a}$
6	$a^{\lvert n\rvert}, \lvert a\rvert < 1$	$\frac{1-a^2}{1-2a\cos\Omega+a^2}$
7	$\cos\Omega_0 n$	$\pi\sum_{k=-\infty}^{\infty}[\delta(\Omega-\Omega_0-2\pi k)+\delta(\Omega+\Omega_0-2\pi k)]$
8	$\sin\Omega_0 n$	$j\pi\sum_{k=-\infty}^{\infty}[\delta(\Omega+\Omega_0-2\pi k)-\delta(\Omega-\Omega_0-2\pi k)]$
9	$(\cos\Omega_0 n)u(n)$	$\frac{e^{j2\Omega}-e^{j\Omega}\cos\Omega_0}{e^{j2\Omega}-2e^{j\Omega}\cos\Omega_0+1}+\frac{\pi}{2}\sum_{k=-\infty}^{\infty}[\delta(\Omega-2\pi k-\Omega_0)+\delta(\Omega-2\pi k+\Omega_0)]$
10	$(\sin\Omega_0 n)u(n)$	$\frac{e^{j\Omega}\sin\Omega_0}{e^{j2\Omega}-2e^{j\Omega}\cos\Omega_0+1}+\frac{\pi}{2j}\sum_{k=-\infty}^{\infty}[\delta(\Omega-2\pi k-\Omega_0)-\delta(\Omega-2\pi k+\Omega_0)]$
11	$\frac{B}{\pi}\operatorname{sinc} Bn, 0 < B < \pi$	$\sum_{k=-\infty}^{\infty}\operatorname{rect}\left(\frac{\Omega-2\pi k}{2B}\right)$
12	$x(n)=\begin{cases}1 & \text{if } \lvert n\rvert \le a\\ 0 & \text{otherwise}\end{cases}$	$\frac{\sin(\Omega[a+\frac{1}{2}])}{\sin(\Omega/2)}$

- Recall the definition of the Fourier transform X of the sequence x:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}.$$

- For all integer k, we have that

$$\begin{aligned} X(\Omega + 2\pi k) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j(\Omega + 2\pi k) n} \\ &= \sum_{n=-\infty}^{\infty} x(n) e^{-j(\Omega n + 2\pi k n)} \\ &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} \\ &= X(\Omega). \end{aligned}$$

- Thus, the Fourier transform X of the sequence x is always 2π-*periodic*.

- If $x_1(n) \stackrel{\text{DTFT}}{\longleftrightarrow} X_1(\Omega)$ and $x_2(n) \stackrel{\text{DTFT}}{\longleftrightarrow} X_2(\Omega)$, then

$$a_1x_1(n) + a_2x_2(n) \stackrel{\text{DTFT}}{\longleftrightarrow} a_1X_1(\Omega) + a_2X_2(\Omega)$$

  where a_1 and a_2 are arbitrary complex constants.
- This is known as the linearity property of the Fourier transform.

- If $x(n) \xleftrightarrow{\text{DTFT}} X(\Omega)$, then
$$x(n-n_0) \xleftrightarrow{\text{DTFT}} e^{-j\Omega n_0} X(\Omega),$$
where n_0 is an arbitrary integer.
- This is known as the translation (or time-domain shifting) property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$e^{j\Omega_0 n}x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega - \Omega_0)$$

where Ω_0 is an arbitrary real constant.

- This is known as the modulation (or frequency-domain shifting) property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$x(-n) \overset{\text{DTFT}}{\longleftrightarrow} X(-\Omega).$$

- This is known as the time-reversal property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$x^*(n) \overset{\text{DTFT}}{\longleftrightarrow} X^*(-\Omega).$$

- This is known as the conjugation property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$x(Mn) \overset{\text{DTFT}}{\longleftrightarrow} \frac{1}{M}\sum_{k=0}^{M-1} X\left(\frac{\Omega - 2\pi k}{M}\right).$$

- This is known as the downsampling property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$(\uparrow M)x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(M\Omega).$$

- This is known as the upsampling property of the Fourier transform.

- If $x_1(n) \overset{\text{DTFT}}{\longleftrightarrow} X_1(\Omega)$ and $x_2(n) \overset{\text{DTFT}}{\longleftrightarrow} X_2(\Omega)$, then

$$x_1 * x_2(n) \overset{\text{DTFT}}{\longleftrightarrow} X_1(\Omega)X_2(\Omega.($$

- This is known as the convolution (or time-domain convolution) property of the Fourier transform.
- In other words, a convolution in the time domain becomes a multiplication in the frequency domain.
- This suggests that the Fourier transform can be used to avoid having to deal with convolution operations.

- If $x_1(n) \overset{\text{DTFT}}{\longleftrightarrow} X_1(\Omega)$ and $x_2(n) \overset{\text{DTFT}}{\longleftrightarrow} X_2(\Omega)$, then

$$x_1(n)x_2(n) \overset{\text{DTFT}}{\longleftrightarrow} \frac{1}{2\pi}\int_{2\pi} X_1(\theta)X_2(\Omega-\theta)\,d\theta.$$

- This is known as the multiplication (or time-domain multiplication) property of the Fourier transform.
- Do not forget the factor of $\frac{1}{2\pi}$ in the above formula!
- This property of the Fourier transform is often tedious to apply (in the forward direction) as it turns a multiplication into a convolution.

- If $x(n) \xleftrightarrow{\text{DTFT}} X(\Omega)$, then

$$nx(n) \xleftrightarrow{\text{DTFT}} j\frac{d}{d\Omega}X(\Omega).$$

- This is known as the frequency-domain differentiation property of the Fourier transform.

- If $x(n) \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega)$, then

$$\sum_{k=-\infty}^{n} x(k) \overset{\text{DTFT}}{\longleftrightarrow} \frac{e^{j\Omega}}{e^{j\Omega}-1} X(\Omega) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k).$$

- This is known as the accumulation (or time-domain accumulation) property of the Fourier transform.

- If $x(n) \xleftrightarrow{\text{DTFT}} X(\Omega)$, then

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(\Omega)|^2 \, d\Omega$$

)i.e., the energy of x and energy of X are equal up to a factor of 2π).

- This is known as Parseval's relation.
- Since energy is often a quantity of great significance in engineering applications, it is extremely helpful to know that the Fourier transform *preserves energy* (up to a scale factor.(